How fast is fast enough to be automatic?

Some educational researchers consider facts to be automatic when a response comes in two or three seconds (Isaacs & Carroll, 1999; Rightsel & Thorton, 1985; Thorton & Smith, 1988). However, performance is not automatic, direct retrieval when it occurs at rates that purposely "allow enough time for students to use efficient strategies or rules for some facts (Isaacs & Carroll, 1999, p. 513)."

Most of the psychological studies have looked at automatic response time as measured in milliseconds and found that automatic (direct retrieval) response times are usually in the ranges of 400 to 900 milliseconds (less than one second) from presentation of a visual stimulus to a keyboard or oral response (Ashcraft, 1982; Ashcraft, Fierman & Bartolotta, 1984; Campbell, 1987a; Campbell, 1987b; Geary & Brown, 1991; Logan, 1988). Similarly, Hasselbring and colleagues felt students had automatized math facts when response times were "down to around 1 second" from presentation of a stimulus until a response was made (Hasselbring et al. 1987)." If however, students are shown the fact and asked to read it aloud then a second has already passed in which case no delay should be expected after reading the fact. "We consider mastery of a basic fact as the ability of students to respond immediately to the fact question. (Stein et al., 1997, p. 87)."

In most school situations students are tested on one-minute timings. Expectations of automaticity vary somewhat. Translating a one-second-response time directly into writing answers for one minute would produce 60 answers per minute. However, some children, especially in the primary grades, cannot write that quickly. "In establishing mastery rate levels for individuals, it is important to consider the learner's characteristics (e.g., age, academic skill, motor ability). For most students a rate of 40 to 60 correct digits per minute [25 to 35 problems per minute] with two or few errors is appropriate (Mercer & Miller, 1992, p.23)." This rate of 35 problems per minute seems to be the lowest noted in the literature.

Other authors noted research which indicated that "students who are able to compute basic math facts at a rate of 30 to 40 problems correct per minute (or about 70 to 80 digits correct per minute) continue to accelerate their rates as tasks in the math curriculum become more complex....[however]...students whose correct rates were lower than 30 per minute showed progressively decelerating trends when more complex skills were introduced. The *minimum* correct rate for basic facts should be set at 30 to 40 problems per minute, since this rate has been shown to be an indicator of success with more complex tasks (Miller & Heward, 1992, p. 100)." Rates of 40 problems per minute seem more likely to continue to accelerate than the lower end at 30.

Another recommendation was that "the criterion be set at a rate [in digits per minute] that is about 2/3 of the rate at which the student is able to write digits (Stein et al., 1997, p. 87)." For example a student who could write 100 digits per minute would be expected to write 67 digits per minute, which translates to between 30 and 40 problems per minute. Howell and Nolet (2000) recommend an expectation of 40 correct facts per minute, with a modification for students who write at less than 100 digits per minute. The number of digits per minute is a percentage of 100 and that percentage is multiplied by 40 problems to give the expected number of problems per minute; for example, a child who can only write 75 digits per minute would have an expectation of 75% of 40 or 30 facts per minute.

If measured individually, a response delay of about 1 second would be automatic. In writing 40 seems to be the minimum, up to about 60 per minute for students who can write that quickly. Teachers themselves range from 40 to 80 problems per minute. Sadly, many school districts have expectations as low as 50 problems in 3 minutes or 100 problems in five minutes. These translate to rates of 16 to 20 problems per minute. At this rate answers can be counted on fingers. So this "passes" children who have only developed procedural knowledge of how to figure out the facts, rather than the direct recall of automaticity.

References

Ashcraft, M. H. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review*, 2, 213-236.

Ashcraft, M. H. & Christy, K. S. (1995). The frequency of arithmetic facts in elementary texts: Addition and multiplication in grades 1 – 6. *Journal for Research in Mathematics Education*, 25(5), 396-421.

Ashcraft, M. H., Fierman, B. A., & Bartolotta, R. (1984). The production and verification tasks in mental addition: An empirical comparison. *Developmental Review*, 4, 157-170.

Ashcraft, M. H. (1985). Is it farfetched that some of us remember our arithmetic facts? *Journal for Research in Mathematics Education*, 16 (2), 99-105.

Campbell, J. I. D. (1987a). Network interference and mental multiplication. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 13 (1), 109-123.

Campbell, J. I. D. (1987b). The role of associative interference in learning and retrieving arithmetic facts. In J. A. Sloboda & D. Rogers (Eds.) *Cognitive process in mathematics: Keele cognition seminars, Vol. 1.* (pp. 107-122). New York: Clarendon Press/Oxford University Press.

Geary, D. C. & Brown, S. C. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in gifted, normal, and mathematically disabled children. *Developmental Psychology*, 27(3), 398-406.

Hasselbring, T. S., Goin, L. T., & Bransford, J. D. (1987). Effective Math Instruction: Developing Automaticity. *Teaching Exceptional Children*, 19(3) 30-33.

Howell, K. W., & Nolet, V. (2000). *Curriculum-based evaluation: Teaching and decision making.* (3rd Ed.) Belmont, CA: Wadsworth/Thomson Learning.

Isaacs, A. C. & Carroll, W. M. (1999). Strategies for basic-facts instruction. *Teaching Children Mathematics*, 5(9), 508-515.

Logan, G. D. (1988). Toward an instance theory of automatization. *Psychological Review*, 95(4), 492-527.

Mercer, C. D. & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education*, 13(3) 19-35.

Miller, A. D. & Heward, W. L. (1992). Do your students really know their math facts? Using time trials to build fluency. *Intervention in School and Clinic*, 28(2) 98-104.

Rightsel, P. S. & Thorton, C. A. (1985). 72 addition facts can be mastered by mid-grade 1. *Arithmetic Teacher*, 33(3), 8-10.

Stein, M., Silbert, J., & Carnine, D. (1997) *Designing Effective Mathematics Instruction: a direct instruction approach* (3rd Ed). Upper Saddle River, NJ: Prentice-Hall, Inc.

Thorton, C. A. & Smith, P. J. (1988). Action research: Strategies for learning subtraction facts. *Arithmetic Teacher*, 35(8), 8-12.